

la ecuación de la recta normal (LN) en un punto (a, b) está dada por:

$$y - b = m_{LN} (x - a)$$

$$y - b = \frac{1}{f'(a)} (x - a)$$

Ejemplo

Sea $y = \text{sen } x$ en el punto $x = \frac{\pi}{6}$ hallar la ecuación de LT y LN.

$$f(x) = \text{sen } x$$

$$f'(x) = y' = \text{cos } x$$

$$\begin{aligned} f'\left(\frac{\pi}{6}\right) &= \text{cos } \frac{\pi}{6} \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

Punto

$$P = (x, f(x))$$

$$= \left(\frac{\pi}{6}, f\left(\frac{\pi}{6}\right)\right)$$

$$= \left(\frac{\pi}{6}, \text{sen } \frac{\pi}{6}\right)$$

$$= \left(\frac{\pi}{6}, \frac{1}{2}\right)$$

Ecuación de la recta Tangente (LT)

$$y - b = f'(a)(x - a)$$

$$y - \frac{1}{2} = \frac{\sqrt{3}}{2} \left(x - \frac{\pi}{6}\right)$$

$$y - \frac{1}{2} = \frac{\sqrt{3}}{2} x - \frac{\sqrt{3}\pi}{12}$$

$$\frac{\sqrt{3}\pi}{12} - \frac{1}{2} = \frac{\sqrt{3}}{2} x - y$$

$$\frac{3(180)}{2} = 3$$

$$\frac{\sqrt{3}\pi - 6}{12} = \frac{\sqrt{3}x - 2y}{2}$$

$$\sqrt{3}\pi - 6 = \frac{12\sqrt{3}x - 24y}{2}$$

$$\sqrt{3}\pi - 6 = \frac{12\sqrt{3}x}{2} - \frac{24y}{2}$$

$$\sqrt{3}\pi - 6 = 6\sqrt{3}x - 12y$$

$$LT: 6\sqrt{3}x - 12y - (\sqrt{3}\pi + 6) = 0$$

Ecuacion de la recta normal

$$m_{LN} = \frac{1}{m_{LT}} = -\frac{1}{\frac{\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}}$$

$$y - b = m_{LN}(x - a)$$

$$y - \frac{1}{2} = -\frac{2}{\sqrt{3}}\left(x - \frac{\pi}{6}\right)$$

$$y - \frac{1}{2} = -\frac{2x}{\sqrt{3}} + \frac{\pi}{3\sqrt{3}}$$

$$LN: \frac{2x}{\sqrt{3}} + y - \left(\frac{1}{2} + \frac{\pi}{3\sqrt{3}}\right) = 0$$