hallar la derivada de $f(x) = \dfrac{x}{1+\sqrt{x}}$

$$f(x) = \frac{x}{1 + x^{\frac{1}{2}}}$$

$$f'(x) = \frac{\left(1+x^{\frac{1}{2}}\right)\cdot 1 - x\left(0+\frac{1}{2}x^{-\frac{1}{2}}\right)}{\left(1+x^{1/2}\right)^2}$$

$$= \frac{1+\sqrt{x} - \frac{1}{2}\sqrt{x}}{\left(1+\sqrt{x}\right)^2} = \frac{2+\sqrt{x}}{\left(1+\sqrt{x}\right)^2}$$

Si $f(x) = (ax+b)\operatorname{sen} x + (cx+d)\cos x$, determinar los valores de las constantes a, b, c, d tales que $f'(x) = x\cos x$

$$f'(x) = \left\{\left[a\operatorname{sen}x + (ax+b)\cos x\right] + \left[c\cos x + (cx+d)(-\operatorname{sen}x)\right]\right\}$$

$$= a\operatorname{sen}x + ax\cos x + b\cos x + c\cos x - cx\operatorname{sen}x - d\operatorname{sen}x$$

$a=1$

$d=1$

$b=0$

$c=0$

NOTA: en este problema necesitamos que
$f'(x) = x\cos x$, es decir
debemos dar valores a a, b, c y d
talque se cumpla que
$f'(x) = x\cos x$

$$\Rightarrow \quad f(x) = x\operatorname{sen}x + \cos x$$

$$f'(x) = \operatorname{sen}x + x\cos x - \operatorname{sen}x$$

$$f'(x) = x\cos x$$

El area de un circulo es πr^2 y su circunferencia $2\pi r$ demostrar que el coeficiente de variación del area respecto al radio es igual a la circunferencia.

$a(r) = \pi r^2$

P.D. $a'(r) = 2\pi r$

$a'(r) = 0 \cdot r^2 + \pi 2r$
$= 2\pi r$

Sea $f(x) = \dfrac{x\,\text{sen}x}{1+x^2}$ calcular $f'(x)$

$$f'(x) = \frac{(1+x^2)(1\cdot\text{sen}x + x\cos x) - x\,\text{sen}x\,(0+2x)}{(1+x^2)^2}$$

$$= \frac{\text{sen}x + x\cos x + x^2\text{sen}x + x^3\cos x - 2x^2\text{sen}x}{(1+x^2)^2}$$

$$= \frac{(\text{sen}x + x^2\text{sen}x - 2x^2\text{sen}x) + (x\cos x + x^3\cos x)}{(1+x^2)^2}$$

$$= \frac{(\text{sen}x - x^2\text{sen}x) + x\cos x\,(1+x^2)}{(1+x^2)^2}$$

$$f'(x) = \frac{\text{sen}x\,(1-x^2) + x\cos x\,(1+x^2)}{(1+x^2)^2}$$

Sea $g(x) = x^4\,\text{sen}x$ hallar $g'(x)$

$g'(x) = 4x^3\,\text{sen}x + x^4\cos x$
$= x^3(4\,\text{sen}x + x\cos x)$

Si $f(x) = 2 + x - x^2$ calcular $f'(0)$, $f'(\tfrac{1}{2})$, $f'(1)$, $f'(-10)$

$f'(x) = 0 + 1 - 2x$
$f'(x) = 1 - 2x$

$f'(0) = 1 - 2(0) = -2$; $f'(\tfrac{1}{2}) = 1 - 2(\tfrac{1}{2}) = 0$; $f'(1) = 1 - 2(1)$, $f'(1) = -1$; $f'(-10) = 1 - 2(-10) = 1 + 20$, $f'(10) = 21$

Sea $f(x) = \sqrt{x}$ hallar $f'(x)$

$$f(x) = x^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2} x^{\frac{1}{2}-1} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2x^{\frac{1}{2}}}$$

$$f'(x) = \frac{1}{2\sqrt{x}}$$

Sea $f(x) = \operatorname{tg} x \sec x$ hallar $f'(x)$

$$f(x) = \frac{\operatorname{sen} x}{\cos x} \cdot \frac{1}{\cos x}$$

$$f(x) = \frac{\operatorname{sen} x}{\cos^2 x}$$

$$f'(x) = \frac{\cos^2 x \cos x - \operatorname{sen} x(-2\cos x \operatorname{sen} x)}{\cos^4 x}$$

$$= \frac{\cos^3 x + 2\operatorname{sen}^2 x \cos x}{\cos^4 x}$$

$$= \frac{\cos^3 x}{\cos^4 x} + \frac{2\operatorname{sen}^2 x}{\cos^2 x} \cdot \frac{\cos x}{\cos^2 x}$$

$$= \sec x + 2\operatorname{tg}^2 x \cdot \sec x$$

$$f'(x) = \sec x\,(1 + 2\operatorname{tg}^2 x)$$