DD MM AA

Pag 421

1. Calcular la derivada de las siguientes funciones reales definidas por.

a) $f(x) = x + \sqrt{x} + \operatorname{sen} x$

$\rightarrow f'(x) = 1 + \frac{1}{2\sqrt{x}} + \cos x$

$\rightarrow f'(x) = 1 + \frac{\sqrt{x}}{2x} + \cos x$ //

b) $f(x) = \operatorname{sen} x + 2\tan x + \sec x$

$\rightarrow f'(x) = \cos x + 2\sec^2 x + \sec x \tan x$ //

$f'(x) = \cos x + \sec x\,(2\sec x + \tan x)$ //

c) $f(x) = \frac{x - \operatorname{sen} x}{\csc x}$

$\rightarrow f'(x) = \frac{\csc x\,(1-\cos x) - (x - \operatorname{sen} x)(-\csc x \cot x)}{\csc^2 x}$

$= \frac{\csc x - \cos x \csc x + x \csc x \cot x - \operatorname{sen} x \csc x \cot x}{\csc^2 x}$

$= \frac{\csc x\,(1 - \cos x + x\cot x - \operatorname{sen} x \cot x)}{\csc^2 x}$

$\rightarrow f'(x) = \frac{1 - \cos x + \cot x\,(x - \operatorname{sen} x)}{\csc x}$ //

d) $f(x) = x + \log x - \frac{1}{x}$

$\Rightarrow f'(x) = 1 + \frac{1}{x}\log e - \frac{0\cdot x - 1\cdot 1}{x^2} = 1 + \frac{1}{x}\log e + \frac{1}{x^2}$

$\Rightarrow f'(x) = 1 + \frac{1}{x}\log e + \frac{1}{x^2}$ //

e) $f(x) = \sqrt{x^1} + \frac{2}{\sqrt[3]{x^1}} = \sqrt{x^1} + 2x^{-\frac{1}{3}}$

$\Rightarrow f'(x) = \frac{1}{2\sqrt{x^1}} + 2\left(-\frac{1}{3}\right)x^{-\frac{1}{3}-1} = \frac{1}{2\sqrt{x^1}} + 2\left(-\frac{1}{3}\right)x^{-4/3}$

$= \frac{\sqrt{x^1}}{2\sqrt{x^1}\sqrt{x^1}} - \frac{2}{3}x^{-4/3} = \frac{\sqrt{x^1}}{2[\sqrt{x^1}]^2} - \frac{2}{3}x^{-4/3}$

$= \frac{\sqrt{x^1}}{2x} - \frac{2}{3}x^{-4/3} = \frac{x^{1/2}}{2x} - \frac{2}{3}x^{-4/3}$

$= \frac{1}{2}x^{\frac{1}{2}-1} - \frac{2}{3}x^{-4/3}$

$\Rightarrow f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - \frac{2}{3}x^{-\frac{4}{3}} = \frac{1}{2\sqrt{x^1}} - \frac{2}{3\sqrt[3]{x^4}}$ //

f) $f(x) = \frac{a+\sqrt{x^1}}{a-\sqrt{x^1}}$ Nota a es constante $\Rightarrow (a)' = 0$

$\Rightarrow f'(x) = \frac{(a-\sqrt{x^1})\left(0+\frac{1}{2\sqrt{x^1}}\right) - (a+\sqrt{x^1})\left(0-\frac{1}{2\sqrt{x^1}}\right)}{(a-\sqrt{x^1})^2}$

$= \frac{\frac{a}{2\sqrt{x^1}} - \frac{\sqrt{x^1}}{2\sqrt{x^1}} - \left(-\frac{a}{2\sqrt{x^1}} - \frac{\sqrt{x^1}}{2\sqrt{x^1}}\right)}{(a-\sqrt{x^1})^2}$

$= \frac{\frac{a}{2\sqrt{x^1}} + \frac{a}{2\sqrt{x^1}}}{(a-\sqrt{x^1})^2} = \frac{\frac{2a}{2\sqrt{x^1}}}{(a-\sqrt{x^1})^2} = \frac{\frac{a}{\sqrt{x^1}}}{(a^2-2a\sqrt{x^1}+x)}$

$\Rightarrow f'(x) = \frac{a}{\sqrt{x}(a^2-2a\sqrt{x^1}+x)} = \frac{a}{a^2\sqrt{x^1}-2ax+x\sqrt{x^1}}$ //

$\Rightarrow f'(x) = \frac{a}{\sqrt{x^1}(a-\sqrt{x^1})^2}$ //

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g) $f(x) = \operatorname{Sen} x \cos x$

$\Rightarrow f'(x) = \cos x \cdot \cos x + \operatorname{sen} x\,(-\operatorname{sen} x)$

$f'(x) = \cos^2 x - \operatorname{sen}^2 x$

Como: $\cos^2 x - \operatorname{sen}^2 x = \cos 2x$

$\Rightarrow f'(x) = \cos 2x$ //

h) $f(x) = \operatorname{sen} x + \log x + \ln x$

$\Rightarrow f'(x) = \cos x + \frac{1}{x}\log e + \frac{1}{x}$; pues $(\log x)' = \frac{1}{x}\log e$

$(\ln x)' = \frac{1}{x}$

$\Rightarrow f'(x) = \cos x + \frac{1}{x}(\log e + 1)$ //

i) $f(x) = \dfrac{ax^2 + bx + c}{\operatorname{sen} x + \cos x}$

$$\Rightarrow f'(x) = \frac{(\operatorname{sen}x + \cos x)(2ax + b) - (ax^2 + bx + c)(\cos x - \operatorname{sen} x)}{(\operatorname{sen} x + \cos x)^2}$$

$$= \frac{(2ax\operatorname{sen}x + 2ax\cos x + b\operatorname{sen}x + b\cos x) - (ax^2 + bx + c)(\cos x - \operatorname{sen}x)}{\operatorname{sen}^2 x + 2\operatorname{sen}x\cos x + \cos^2 x}$$

$$= \frac{2ax\operatorname{sen}x + 2ax\cos x + b\operatorname{sen}x + b\cos x - (ax^2\cos x + bx\cos x + c\cos x - ax^2\operatorname{sen}x - bx\operatorname{sen}x - c\operatorname{sen}x)}{(\operatorname{sen}^2 x + \cos^2 x) + 2\operatorname{sen}x\cos x}$$

$$= \frac{2ax\operatorname{sen}x + 2ax\cos x + b\operatorname{sen}x + b\cos x - ax^2\cos x - bx\cos x - c\cos x + ax^2\operatorname{sen}x + bx\operatorname{sen}x + c\operatorname{sen}x}{1 + \operatorname{sen}2x}$$

$$= \frac{ax^2\operatorname{sen}x + 2ax\operatorname{sen}x + bx\operatorname{sen}x + b\operatorname{sen}x + c\operatorname{sen}x) - (ax^2\cos x - 2ax\cos x + bx\cos x - b\cos x + c\cos x)}{1 + \operatorname{sen}2x}$$

$$f'(x) = \frac{\operatorname{sen}x\,[ax^2 + (2a+b)x + b + c] - \cos x\,[ax^2 + (b-2a)x + c - b]}{1 + \operatorname{sen}2x}$$ //

j) $f(x) = \frac{1}{2}\csc x\,(3 - 4\operatorname{sen}x + \sqrt{x})$

$$f(x) = \frac{3}{2}\csc x - 2\operatorname{sen}x\csc x + \frac{1}{2}\sqrt{x}\csc x$$

$$\Rightarrow f'(x) = \frac{3}{2}(-\csc x\cot x) - 2\left[\cos x\csc x + \operatorname{sen}x(-\csc x\cot x)\right] + \frac{1}{2}\left[\frac{1}{2\sqrt{x}}\csc x + \sqrt{x}(-\csc x\cot x)\right]$$

$$= -\frac{3}{2}\csc x\cot x - 2\cos x\csc x + 2\operatorname{sen}x\csc x\cot x + \frac{1}{4\sqrt{x}}\csc x + \frac{\sqrt{x}}{2}(-\csc x\cot x)$$

$$= -\frac{3}{2}\csc x\cot x - 2\cos x\csc x + 2\operatorname{sen}x\csc x\cot x + \frac{1}{4\sqrt{x}}\cos x - \frac{\sqrt{x}}{2}\csc x\cot x$$

$$= -\frac{3}{2}\csc x\cot x - 2\cos x\csc x + 2\operatorname{sen}x\csc x\frac{\cos x}{\operatorname{sen}x} + \frac{1}{4\sqrt{x}}\cos x - \frac{\sqrt{x}}{2}\csc x\cot x$$

$$= -\frac{3}{2}\csc x\cot x + \frac{1}{4\sqrt{x}}\cos x - \frac{\sqrt{x}}{2}\csc x\cot x$$

$$= \frac{1}{2}\left[\frac{1}{2\sqrt{x}}\cos x - 3\csc x\cot x - \sqrt{x}\csc x\cot x\right]$$

$$\Rightarrow f'(x) = \frac{1}{2}\left[\frac{1}{2\sqrt{x}}\cos x - (3+\sqrt{x})\csc x\cot x\right] \;//$$

NOTA La respuesta del Libro es igual a esta simplemente se sigio operando para obtener una expresión mas simple.

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l) $f(x) = +x^{1/2} + x^{1/3} + x^{1/4} + x^{1/5}$, $x>0$

$\Rightarrow f'(x) = +\frac{1}{2}x^{\frac{1}{2}-1} + \frac{1}{3}x^{\frac{1}{3}-1} + \frac{1}{4}x^{\frac{1}{4}-1} + \frac{1}{5}x^{\frac{1}{5}-1}$

$\Rightarrow f'(x) = +\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{3}x^{-\frac{2}{3}} + \frac{1}{4}x^{-\frac{3}{4}} + \frac{1}{5}x^{-\frac{4}{5}}$ //

n) $f(x) = \operatorname{sen}^3 x = \operatorname{sen} x \operatorname{sen}^2 x = \operatorname{sen}x(\operatorname{sen}x\operatorname{sen}x)$

$\Rightarrow f'(x) = \cos x \operatorname{sen}^2 x + \operatorname{sen}x(\cos x \operatorname{sen}x + \operatorname{sen}x\cos x)$

$= \cos x \operatorname{sen}^2 x + \cos x \operatorname{sen}^2 x + \cos x \operatorname{sen}^2 x$

$= 3\cos x \operatorname{sen}^2 x$

$\Rightarrow f'(x) = 3\operatorname{sen}^2 x \cos x$

2.- Calcular $\frac{dy}{dx}$ si:

a) $y = x\operatorname{sen}x + \cos x$

$\Rightarrow \frac{dy}{dx} = (\operatorname{sen}x + x\cos x) - \operatorname{sen}x$

$\frac{dy}{dx} = x\cos x$ //

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b) $y = \dfrac{1+\cos x}{1-\cos x}$

$$\Rightarrow \frac{dy}{dx} = \frac{(1-\cos x)(0+(-\operatorname{sen} x)) - (1+\cos x)(0-(-\operatorname{sen} x))}{(1-\cos x)^2}$$

$$= \frac{(1-\cos x)(-\operatorname{sen} x) - (1+\cos x)(\operatorname{sen} x)}{(1-\cos x)^2}$$

$$= \frac{-\operatorname{sen} x + \operatorname{sen} x \cos x - (\operatorname{sen} x + \operatorname{sen} x \cos x)}{(1-\cos x)^2}$$

$$= \frac{-2\operatorname{sen} x}{(1-\cos x)^2}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{2\operatorname{sen} x}{(1-\cos x)^2} \quad //$$

c) $y = 2x\cos x + (x^2-2)\operatorname{sen} x$

$$y = 2x\cos x + x^2 \operatorname{sen} x - 2\operatorname{sen} x$$

$$\Rightarrow \frac{dy}{dx} = 2(\cos x + x(-\operatorname{sen} x)) + (2x\operatorname{sen} x + x^2\cos x) - 2\cos x$$

$$= 2\cos x - 2x\operatorname{sen} x + 2x\operatorname{sen} x + x^2\cos x - 2\cos x$$

$$\frac{dy}{dx} = x^2\cos x \quad //$$

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3. Sea $f(x) = 2 + x + x^2 - x^3$, Calcular $f'(0)$, $f'\left(\frac{1}{4}\right)$ y $f'(-10)$

$$f'(x) = 0 + 1 + 2x - 3x^2$$

$$f'(x) = 1 + 2x - 3x^2$$

$$\Rightarrow f'(0) = 1 + 2(0) - 3(0)$$

$$f'(0) = 1 \quad //$$

$$\Rightarrow f'\left(\frac{1}{4}\right) = 1 + 2\left(\frac{1}{4}\right) - 3\left(\frac{1}{4}\right)^2$$

$$= 1 + \frac{1}{2} - \frac{3}{16} = \frac{16 + 8 - 3}{16}$$

$$f'\left(\frac{1}{4}\right) = \frac{21}{16} \quad //$$

$$\Rightarrow f'(-10) = 1 + 2(-10) - 3(-10)^2$$

$$= 1 - 20 - 300$$

$$f'(-10) = -319 \quad //$$